How Our Intuition Deceives Us!

- Some Counter-intuitive Probability Problems

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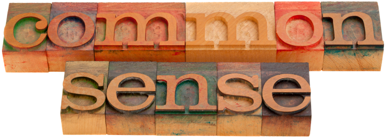


"Mathematics is the extension of common sense by other means."

"But common sense is almost always misleading in mathematics."

Armoring U.S. Airplanes

How to Armor U.S. Airplanes in World War II

As an engineer of the Air Force, you job is to design the optimal armoring system for the air planes:

- Armor too little: planes easily get shot down by enemy fighters;
- Armor too much: planes are too heavy, less maneurerable and use more fuel.

How to Armor U.S. Airplanes in World War II

During World War II, when American planes came back from engagements over Europe, they were covered in bullet holes that were distributed as below:

Section of plane	Bullet holes per square foot
Engine	0.81
Fuel system	1.78
Wings	2.01
Rest of plane	1.53

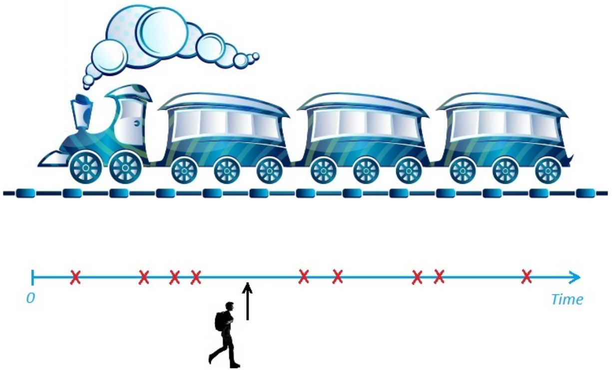
Question: Where to armor on the plane?

Prof. Random and NYC Subway

Professor Random and the NYC Subway

In NYC, Professor Random at Columbia University takes subway train #1 twice a day:

- once in the morning (from his apartment at 42nd st. to his office at 116th st.);
- the other in the afternoon (from office back to home).



Professor Random and the NYC Subway

Suddenly Prof. Random decides to estimate the average interarrival time of the NYC subway.
He proposes two approaches:

(i) He takes a day off (if his boss, the department head Prof. Harsh, allows so) and sit on the platform. He records these times within a day and take the average.

(ii) He does not take a day off (say Prof. Harsh may be unhappy). Since He takes subway twice a day, he records lengths of interarrival times that covers his arrival time. For instance, suppose he arrives at the platform at 8:00am, a staff tells me the previous train left at 7:50am and he finds the next subway arrives at 8:05am and he gets on it. In this way he gets one data point: 15 minutes. He does this for a whole month and get 60 data points (almost for free), then he averages them.

Which one provides an unbiased estimate? Why?

Sheldon's Psychic Power

Sheldon's Psychic Power

Professor Sheldon Cooper, a famous theoretical physicist at Caltech, has an IQ of 187. Sheldon claims that he can alter the outcome of coin tosses using his psychic power. His roommate, a experimental physicist Prof. Leonard Hofstadter, who has an IQ of 173, doesn't believe him. The two high-IQ scientists had an argument.

Sheldon's Psychic Power

To settle this argument, their common friend, Mr. Howard Wolowitz, who is an Aerospace Engineer at Caltech, proposes to design an experiment.
A fair coin is tossed for 10,000 times and the number of *heads* is recorded. How likely are these outcomes if Sheldon does not have a super power?

1. 5,000
2. 5,012
3. 5,113
4. 5,562
5. 5,972

Given Case 4 is observed, is it safe to conclude that Sheldon indeed has a super power?

The Pattern Problem

The Pattern Problem

Consider successive independent flips of a fair coin, i.e., it has an equal probability of landing heads (H) and tails (T).

A given segment of finitely many consecutive outcomes is called a *pattern*. The pattern is said to occur at the $n^{\rm th}$ flip if the pattern is completed a at flip n.

For example, the pattern $HTHH$ occurs at step 11, 14 and 22 in the sequence

$$HHTHTTT\underline{HTH\underline{H}}\underline{THH}HTTT\underline{HTHH}TTH\ldots$$

and at no other times among the first 25 flips. Note that the pattern occurrences at 11 and 14 overlap, which is allowed.

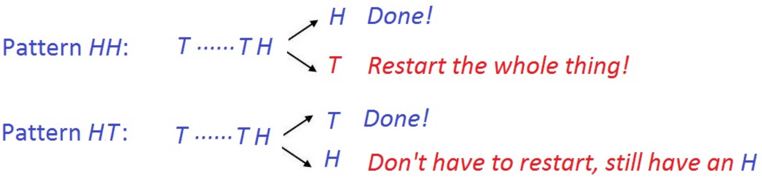
The Pattern Problem

Now consider the following four patterns:

$$\mathcal{A} \equiv HH, \quad \mathcal{B} \equiv HT, \quad \mathcal{C} \equiv TT \quad and \quad \mathcal{D} \equiv TH.$$

1. Which one "on average" appears in the smallest number of flips?
2. Starting with an existing pattern, which one "on average" appears again in the smallest number of flips?

The Pattern Problem



Berkeley University's Gender Bias Case

Berkeley University's Gender Bias Case

In 1973, University of California, Berkeley was sued for bias against women who had applied for admission to graduate schools there. The admission figures for the fall of 1973 showed that men applying were more likely than women to be admitted, and the difference was so large that it was unlikely to be due to chance.

	Applicants	Admitted
Men	8442	**44%**
Women	4321	35%

Berkeley University's Gender Bias Case

But when examining the individual departments, it appeared that no department was significantly biased against women. In fact, most departments had a "small but statistically significant bias in favor of women." The data from the six largest departments are listed below.

Department	Men		Women	
	Applicants	Admitted	Applicants	Admitted
A	825	62%	108	**82%**
B	560	63%	25	**68%**
C	325	**37%**	593	34%
D	417	33%	375	**35%**
E	191	**28%**	393	24%
F	272	6%	341	**7%**

Question: How to explain?

Berkeley University's Gender Bias Case

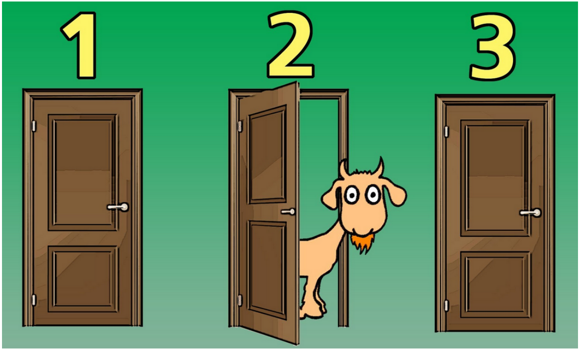
Answer: Women tended to apply to competitive departments with low rates of admission even among qualified applicants (such as in the English Department), whereas men tended to apply to less-competitive departments with high rates of admission among the qualified applicants (such as in engineering and chemistry).

The Monte Hall Problem

The Monte Hall Problem

Suppose you're on a game show, and you're given the choice of three doors:

- Behind one door is a car;
- Behind the others, goats.



The Monte Hall Problem

- You pick a door, say No. 1;
- The host, who knows what's behind the doors, opens another door, say No. 2, which has a goat. He then says to you, "Do you want to pick door No. 3?"

Question: Is it to your advantage to switch your choice?

Solution:

Behind door 1	Behind door 2	Behind door 3	Result if staying at door #1	Result if switching to the door offered
Car	Goat	Goat	**Wins Car**	Wins Goat
Goat	**Car**	Goat	Wins Goat	**Wins Car**
Goat	Goat	**Car**	Wins Goat	**Wins Car**

More Fun Probability Problems