Course Syllabus

Course Description
This is a Ph.D. level course in stochastic models designed to develop a solid understanding of uncertain phenomena and mathematical tools used to model and analyze random observations in industrial engineering. The course will provide both rigorous proof-based mathematical basis and related applications.

Office Hours
(a) Yunan Liu (Instructor)
   446 Daniels, Thursday 3:00 - 5:00 pm
   Email: yunan_liu@ncsu.edu
   Website: http://yunanliu.wordpress.ncsu.edu

(b) Kyle Hovey (Teaching Assistants)
   444 Daniels, Monday 2:00 - 4:00 pm
   Email: kahovey@ncsu.edu

Course Website
http://moodle.wolfware.ncsu.edu

Prerequisites
Knowledge on probability theory and stochastic models, such as ISE589.

Reference Texts
(a) Required:

(b) Recommended:

Homework
There will be weekly assignments due every Tuesday in class. Graded assignments will be returned in class.

- Students are encouraged to collaborate with other students in the class, as long as each person writes his/her own solutions.

- But any such collaboration should be clearly noted (If some ideas of your solutions come from the discussion with another person, write his/her name on your solution).

- Copying homework from another student (past or present) is forbidden.

- Late homework will NOT be accepted.
Recitations

479 Daniels, Monday 2:00-3:00 (as part of the TA office hours). The TA helps solve selected homework problems.

Exams

All exams are in class, closed book, closed note. You are allowed to bring a two-sided cheat sheet.

- 1st midterm: September 29 (Thursday), 9:00 - 11:30.
- 2nd midterm: November 1 (Tuesday), 9:00 - 11:30.
- Final: December 8 (Tuesday), 8:00 - 11:00.

Project

There will be a project (done by a group of at most two students) which consists of two parts:

(i) Modeling real systems: Apply mathematical methods to model a real system that you encounter in your daily life (e.g., bank, highway, gym, etc.) Explain why your model is appropriate; propose methods to help improve the operational efficiency of this system; and conduct some analysis (numerical or analytic).

(ii) Popularizing OR methods and results: Choose \( \geq 5 \) results in your OR courses (e.g., 760, 505, 723, etc.) and explain them in plain and comprehensible words. The goal is to make non-OR people (such as your dad, assuming he is not a math professor!) understand them. This will help improve your teaching skills. Albert Einstein said: “If you cannot explain it simply, you do not understand it well enough!”

The project will be due by the end of the term.

Grading

Define the following random variables:

\[ HW \equiv \text{homework}, \quad F \equiv \text{project}, \quad M_1 \equiv \text{midterm 1}, \quad M_2 \equiv \text{midterm 2}, \quad F \equiv \text{final exam} \quad \text{and} \quad G \equiv \text{overall grade}. \]

Then the overall grade is given by

\[ G \equiv HW \times 15\% + P \times 10\% + M_1 \times 25\% + M_2 \times 25\% + F \times 30\% - \min(M_1, M_2, F) \times 5\%. \]

Tentative Course Outline

1. Review of Probability Theory
   - Probability space
   - Independence and dependence
   - Conditional probability and Bayes’ formula
   - Random variables: definition, distribution functions, discrete and continuous types
   - Random variables: expectation, variance, covariance and moment generating functions
   - Markov’s inequality and Chebyshev’s inequality
• Modes of convergence
• Limit theorems: strong law of large number (SLLN) and central limit theorem (CLT)

2. Discrete-Time Markov Chain (DTMC)
• Definition: the Markov property
• Classification of states: transience and recurrence
• Chapman-Kolmogorov equations
• The Gambler’s ruin problem
• Steady-state distributions
• DTMCs with absorbing states/classes: canonical forms, fundamental matrices, and mean times until absorption
• Time reversibility, random walk on a graph

3. Poisson Process (PP)
• Exponential distribution: the lack-of-memory property and its applications
• Equivalency of the three definitions of Poisson processes
• Properties of Poisson: independent thinning and superposition
• Order statistics and conditional distributions of the arrival times
• Generalization 1: compound Poisson process (thinning and superposition for NPP)
• Generalization 2: nonhomogeneous Poisson process (definitions, properties and connection to PP)
• The $M_t/G/\infty$ queue: number of customers at time $t$ and the departure process

4. Continuous-Time Markov Chain (CTMC)
• CTMC: basic definition, transition probability and rate matrices
• Kolmogorov-Chapman equation and Kolmogorov ODE
• Steady state: two different approaches
• Birth-and-death processes and Markovian queueing networks
• Time reversibility

5. Renewal Counting Process (RCP)
• Renewal functions and renewal equations
• Renewal reward processes (RRP)
• Limit theorems for RCP and RRP
• Age, excess and spread of an RCP
• An application: patterns