ISE 760 **Assignment 2**
Applied Stochastic Models **August 9, 2018**

Assignment 2: Review of Probability (continued)

– due September 6

Do the following exercises in Green Ross:

- 2.37,
- 2.50,
- 2.78(b) for 11th ed. (2.68(b) for 9th or 10th ed.)

Other problems:

1. Let $X_1, \ldots, X_n$ be a sequence of I.I.D. random variables, $\mathbb{E}[X_1] = \mu$, $Var(X_1) = \sigma^2$. Let the *sample mean* $\bar{X}_n$ and *sample variance* S_n^2 be defined as:

$$\bar{X}_n \equiv \frac{1}{n}\sum_{i=1}^{n} X_i,$$
$$S_n^2 \equiv \frac{1}{n-1}\sum_{i=1}^{n} \left(X_i - \bar{X}_n\right)^2.$$

   Compute $\mathbb{E}[\bar{X}_n]$ and $\mathbb{E}[S_n^2]$.

2. (CLT) Fifty numbers are rounded off to the nearest integer and then summed. If the individual roundoff errors are uniformly distributed between -0.5 and 0.5, what is the approximate probability that the resultant sum differs form the exact sum by more than 3?

3. (Law of Small Numbers) Consider a sequence of random variables $X_1, X_2, \ldots$, where X_n has a binomial distribution with parameter (n, p_n), $p_n = \lambda/n$, i.e.,

$$\mathbb{P}(X_n = k) = \left(\begin{array}{c} n \\ k \end{array}\right) p_n^k (1-p_n)^{n-k} = \frac{n!}{k!(n-k)!} p_n^k (1-p_n)^{n-k}.$$

   (a) Show that the limiting distribution of X_n is Poisson with mean λ, i.e., show that

$$\mathbb{P}(X_n = k) \to \mathbb{P}(X_\infty = k) = e^{-\lambda}\frac{\lambda^k}{k!}, \quad k \geq 0,$$

   where X_∞ is a Poisson random variable with mean λ.
   Hint: Use the fact that $\lim_{n\to\infty}(1+1/n)^n = e$ *and* $\lim_{n\to\infty}(1-1/n)^n = e^{-1}$

   (b) Solve Exercise 2.32 in Green Ross using your result in (a).

Reading: On Chapter 3, read Section 3.2-3.5. There are a lot of good examples that can help you understand conditional probability, but some are tricky. Make sure you at least completely understand Example 3.11, 3.19, 3.20, 3.21. Read other examples if you have time. If you want to do more exercises, do those with stars: they have answers in the back of the book.