Last Assignment: *Due on November 27*

Renewal Counting Processes

From Green Ross: Do Exercise 7.21, 7.26, 7.38, 7.41 in Chapter 7.

Extra part for 7.21: Instead of having a $PP(\lambda)$ arrival process, suppose the arrival process is a renewal process having interarrival CDF F. Would the number of events by time t constitue a (poissibly delayed) renewal process if an event corresponds to a customer

(a) entering the bank? (b) leaving the bank?

Repeat (a) and (b) assuming F is exponential.

Other Problems:

1. (**Patterns**) Consider successive flips of a fair coin.
   (a) Compute the mean number of flips until the pattern **TTHTTHTT** appears.
   (b) Which pattern requires a larger expected time to occur: **HHTT**, **HTHT**, or **HHHH**?
2. (**More Patterns**) Consider successive flips of a coin having probability p of landing heads. Find the expected number of flips until the following sequences appear:
   (a) $\mathcal{A} =$ **HHTTHH**.
   (b) $\mathcal{B} =$ **HTHTT**.

   Suppose now $p = 2/3$.

   (c) Find $\mathbb{P}(\mathcal{A}$ occurs before $\mathcal{B})$.
   (d) Find the expected number of flips until either $\mathcal{A}$ or $\mathcal{B}$ occurs.

Reading: Read Green Ross Sections 7.1-7.4, 7.7 and 7.9. Examples 7.12, 7.13, 7.23, 7.24 and 7.25 are good examples which are very representative. If you need more exercises, you can also study Exercise 7.8, 7.22, 7.25 and 7.42 which have answers in the back.

Appendix: Alternating Renewal Processes (ARP)

Consider a system that can be in one of two states: *on* or *off*. Initially it is on, and it remains on for a time Z_1; it then goes off and remains off for a time Y_1. It then goes on for a time Z_2; then off for a time Y_2; then on, and so on. If the two sequences $Z_1, Z_2, \dots$ and $Y_1, Y_2, \dots$ are I.I.D. non-negative r.v.'s, the process which keeps track of the system status for all time is called ARP.

Suppose we are concerned with P_{on}, *the long-run proportion of time that the system is on.* We let $X_1, X_2, \dots$ be interarrival times, where $X_n \equiv Y_n + Z_n$, and let $\{N(t), t \geq 0\}$ be an RCP defined w.r.t. these interarrival times. Suppose reward is earned with rate 1 (per unit of time) whenever the system is *on* and no reward is earned when the system is *off*. Let $R(t)$ be the total reward earned by t. Then we can compute P_{on} as below:

$$P_{on} = \lim_{t\to\infty} \frac{R(t)}{t} = \frac{\text{mean reward in one cycle}}{\text{mean cycle length}} = \frac{\mathbb{E}[Z]}{\mathbb{E}[Y] + \mathbb{E}[Z]},$$

where the **second** equality holds by the SLLN of RRP introduced and proved in class.

Hints

7.21: The original part is easy! Define the system is on when the server is busy, then this is an application of ARP (see the definition at the end of this HW). Relate this problem to Problem 2 of IC-PS-13. The extra part is not too bad either.

7.26: Easy! Apply SLLN for RRP. On the modeling, you now have to aggregate Parts (a) and (b) of Problem 3 in ICPS-13.

7.38: Apply SLLN for RRP. The key is to define the right cycle.

7.41: Easy! This is an application of Problem 1 Part (d) in ICPS-14.

OP 1: Easy! Follow our analysis in ICPS-14.

OP 2: Easy! Follow our analysis in ICPS-14.