Assignment 1: Introduction to Simulation; Probability Review

Due on July. 11

1. Use simulation to approximate the following integrals.

$$(a)\ \int_{-2}^{2} e^{x+x^2}dx, \qquad (b)\ \int_{-\infty}^{\infty} e^{-x^2}dx, \qquad (c)\ \int_0^1\int_0^1 e^{(x+y)^2}dydx$$

2. The following data yield the arrival times and service times that each customer will require, for the first 13 customers at a single server system. Upon arrival, a customer either enters service if the server is free or joins the waiting line. The system is operated under the *first-come first-served* (FCFS) rule, that is, when the server completes work on a customer, the next one in line (i.e. the one who has been waiting the longest) enters the service. Let $Q(t)$ denote the total number of customers in the system at t.

Arrival Times:	12	31	63	95	99	154	198	221	304	346	411	455	537
Service Times:	40	32	55	48	18	50	47	18	28	54	40	72	12

   (a) Determine the departure times of these 13 customers and plot a sample path of $Q(t)$ (either by hand or by computer). See the sample path on p.13 of the class notes "Introduction to simulations" for example.

   (b) Repeat (a) when there are two servers and a customer can be served by either one.

   (c) Repeat (a) under the new assumption of *last-come first-served* (LCFS), that is, when the server completes a service, the next customer to enter service is the one that has been waiting the least time.

3. Use the following data

Arrival Times:	1	5	13	14	21	26	29
Service Times:	6	8	5	6	4	6	7

   (a) to determine the departure times of customers in a single server queue, and

   (b) to draw a sample path of the total number of customers in the system

   under the assumption that all customers present are simultaneously in service, and each receives the same fraction of the server capacity. For example, if at time t there is only one customer who has 3 min of time left to complete his service and a second customer arrives whose service requirement is 4 min, then (assuming no other customers arrive until these two customers leave) it will take 6 min for the first customer, and $6 + 1 = 7$ min for the second customer, to complete their services (there are two customers, so their processing times double) . This discipline is known as *processor sharing* (PS) and is commonly used in modeling computer and communication networks.

4. A certain component is critical to the operation of an electrical system and must be replaced immediately upon failure. If the mean lifetime of this type of component is 100 hours and its standard deviation is 30 hours, how many of the components must be in stock so that the probability that the system is in continual operation for the next 2000 hours is at least 0.95?

5. Suppose that 10 observations from a distribution with unknown mean μ

$$7.3,\ 6.1,\ 3.8,\ 8.4,\ 6.9,\ 7.1,\ 5.3,\ 8.2,\ 4.9,\ 5.8.$$

   Compute $\bar{X}(10)$, $S^2(10)$, and an approximate 95 percent confidence interval for μ.

6. A pair of dice are to be continually rolled until all the possible outcomes $2, 3, \ldots, 12$ have occurred at least once. Develop a Monte-Carlo simulation experiment to estimate N: the expected number of dice rolls that are needed. Implement your algorithm in MatLab/Python to give the 95% confidence interval for $\mathbb{E}[N]$ with $n = 100,000$.

7. Consider a triangle with height h and a point uniformly chosen within the triangle. Let X be the distance from the point to the base of the triangle. Find the CDF and the PDF of X.