Course Syllabus

Course Description

This is a Ph.D. level course in stochastic models and queueing theory designed to develop a solid understanding of uncertain phenomena and mathematical tools used to model and analyze random observations in industrial engineering and operations research. The course will provide both rigorous proof-based mathematical basis and related applications.

Instructor

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Teaching assistents

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Course website

https://yunanliu.wordpress.ncsu.edu/neu-summer-course/

Prerequisites

Knowledge on probability theory and stochastic models.

Reference Texts

(α) *Required:*
Ross, S. M. *Intro to Probability Models.* 10th or 11th Edition, Academic Press, Elsevier.

(β) *Recommended:*
Ross, S. M. *Stochastic Processes.* 2nd Edition, Academic Press, Weley, 1996.

Tentative Course Outline

1. Review of Probability Theory
    - Probability space
    - Independence and dependence
    - Conditional probability and Bayes' formula
    - Random variables: definition, distribution functions, discrete and continuous types

- Random variables: expectation, variance, covariance and moment generating functions
- Markov's inequality and Chebyshev's inequality
- Modes of convergence
- Limit theorems: strong law of large number (SLLN) and central limit theorem (CLT)

2. Discrete-Time Markov Chain (DTMC)

- Definition: the Markov property
- Classification of states: transience and recurrence
- Chapman-Kolmogorov equations
- The Gambler's ruin problem
- Steady-state distributions
- DTMCs with absorbing states/classes: canonical forms, fundamental matrices, and mean times until absorption
- Time reversibility, random walk on a graph

3. Poisson Process (PP)

- Exponential distribution: the lack-of-memory property and its applications
- Equivalency of the three definitions of Poisson processes
- Properties of Poisson: independent thinning and superposition
- Order statistics and conditional distributions of the arrival times
- Generalization 1: compound Poisson process (thinning and superposition for NPP)
- Generalization 2: nonhomogeneous Poisson process (definitions, properties and connection to PP)
- The $M_t/G/\infty$ queue: number of customers at time t and the departure process

4. Continuous-Time Markov Chain (CTMC)

- CTMC: basic definition, transition probability and rate matrices
- Kolmogorov-Chapman equation and Kolmogorov ODE
- Steady state: two different approaches
- Birth-and-death processes and Markovian queueing networks
- Time reversibility

5. Renewal Counting Process (RCP)

- Renewal functions and renewal equations
- Renewal reward processes (RRP)
- Limit theorems for RCP and RRP
- Age, excess and spread of an RCP
- An application: patterns