1. **Songchen’s Barbershop in Raleigh (An $M/M/1/3 + M$ Queue)**
   Yang Songchen operates a small barbershop by herself in Raleigh with room for at most three customers, one in service and two waiting.

   - Potential customers arrive according to a $PP(\lambda)$ with $\lambda = 10$ per hour. If a potential arrival finds the barber shop full, with a customer in service and two other customers waiting, he (she) will leave and will not affect future arrivals;
   - Successive service times are I.I.D. $\text{Exp}(\mu)$ r.v.’s with mean $1/\mu = 30$ minutes;
   - Waiting customers have limited patience, with each waiting customer being willing to wait only a random amount of time, if the customer has not started service by that time, the customer will abandon, leaving without receiving service. Customers’ patience times are I.I.D. r.v.’s following $\text{Exp}(\gamma)$, with mean $1/\gamma = 20$ minutes.

   (a) What is the long-run proportion of time Songchen is busy?
(b) What is the long-run proportion of time the shop is full?

(c) What is the long-run average number of customers in the shop?

(d) What is the long-run proportion of customers that are served (among all potential arrivals)?